```
/*
wxMaxima 0.7.3a http://wxmaxima.sourceforge.net
Maxima 5.13.0 http://maxima.sourceforge.net
Using Lisp GNU Common Lisp (GCL) GCL 2.6.8 (aka GCL)
Distributed under the GNU Public License. See the file COPYING.
Dedicated to the memory of William Schelter.
This is a development version of Maxima. The function bug_report()
provides bug reporting information.
/*
Clear all symbols


(%i1) Kill(all)$
/*
Definitions (1) - (3)

Relabel variables since maxima offers no direct support for greek letters
alpha: a
rho: r
mu: m


(%i2) profit(w,p):= p*a - w -c;
(%o2) profit(w, p):= p a - w - c

(%i3) r:q*(1+m*log(w/W));
(%o3) q (m log(w/W) + 1)

(%i4) a:r*x + (1-r)*y;
(%o4) y (1 - q (m log(w/W) + 1)) + q x (m log(w/W) + 1)
/*
Zero profit price (6)

Set w=W

(%i5) w:W;
(%o5) W

(%i6) solve([profit(w,p)=0],p);
                                  W + c
(%o6) [ p = - ------------------------------- ]
              (q - 1) y - q x
/*
Clear w=W
```

```
(%i7)  kill(W,w);
(%o7)  done
/*
Check equation (6)

(%i8)  -(W+c)/((q-1)*y-q*x) = (W+c)/(q*(x-y)+y);
```

$$(\%o8)\quad \frac{-W-c}{(q-1)y-qx}=\frac{W+c}{y+q(x-y)}$$

```
/*
Check if both expressions are equal after simplifying

(%i9)  is(ratsimp(%));
(%o9)  true
/*
Selection wage curve (7) - (9)

(%i10)  W:w;
(%o10)  w

(%i11)  Dprofit(w,p):= diff(profit(w,p),w);
(%o11)  Dprofit(w,p):=diff(profit(w,p),w)

(%i12)  D2profit(w,p):=diff(profit(w,p),w,2);
(%o12)  D2profit(w,p):=diff(profit(w,p),w,2)

(%i13)  Dprofit(w,p) = (p*q*m*(x-y)*1/w-1);
```

$$(\%o13)\quad p\left(\frac{mqx}{w}-\frac{mqy}{w}\right)-1=\frac{mpq(x-y)}{w}-1$$

```
(%i14)  is(ratsimp(%));
(%o14)  true

(%i15)  D2profit(w,p) = (-p*q*m*(x-y)*1/(w^2));
```

$$(\%o15)\quad p\left(\frac{mqy}{w^2}-\frac{mqx}{w^2}\right)=-\frac{mpq(x-y)}{w^2}$$

```
(%i16)  is(ratsimp(%));
(%o16)  true
```

```
(%i17) solve([Dprofit(w,p)=0],p);
```

$$(\%o17)\quad \left[p = -\frac{w}{m\,q\,y - m\,q\,x} \right]$$

```
(%i18) -w/(m*q*y-m*q*x) = w/(m*q*(x-y));
```

$$(\%o18)\quad -\frac{w}{m\,q\,y - m\,q\,x} = \frac{w}{m\,q\,(x - y)}$$

```
(%i19) is(ratsimp(%));
(%o19) true
/*
Equilibrium (10), (11)

(%i20) solve([profit(w,p)=0, Dprofit(w,p)=0],[w,p]);
```

$$(\%o20)\quad \left[\left[w = -\frac{c\,m\,q\,y - c\,m\,q\,x}{((m - 1)q + 1)y + (1 - m)q\,x},\ p = \frac{c}{((m - 1)q + 1)y + (1 - m)q\,x} \right] \right]$$

```
/*
Check w

(%i21) -(c*m*q*y-c*m*q*x)/(((m-1)*q+1)*y+(1-m)*q*x) =
 (m*q*(x-y))/(q*(x-y)*(1-m)+y)*c;
```

$$(\%o21)\quad \frac{c\,m\,q\,x - c\,m\,q\,y}{((m - 1)q + 1)y + (1 - m)q\,x} = \frac{c\,m\,q\,(x - y)}{y + (1 - m)q\,(x - y)}$$

```
(%i22) is(ratsimp(%));
(%o22) true
/*
Check p

(%i23) c/(((m-1)*q+1)*y+(1-m)*q*x) = c/(q*(x-y)*(1-m)+y);
```

$$(\%o23)\quad \frac{c}{((m - 1)q + 1)y + (1 - m)q\,x} = \frac{c}{y + (1 - m)q\,(x - y)}$$

```
(%i24) is(ratsimp(%));
(%o24) true
/*
Stability (12)

(%i25) kill(w,W);
```

(%o25) done

(%i26) solve([p*q*m*(x-y)*1/w-1=0],w);

(%o26) [w = m p q x - m p q y]

(%i27) p:(W+c)/(q*(x-y)+y);

(%o27) $$\frac{W+c}{y+q(x-y)}$$

(%i28) w:m*p*q*x-m*p*q*y;

(%o28) $$\frac{m\,q\,x(W+c)}{y+q(x-y)}-\frac{m\,q\,y(W+c)}{y+q(x-y)}$$

(%i29) w = (m*(x-y)*q)/((x-y)*q+y)*(W+c);

(%o29) $$\frac{m\,q\,x(W+c)}{y+q(x-y)}-\frac{m\,q\,y(W+c)}{y+q(x-y)}=\frac{m\,q(x-y)(W+c)}{y+q(x-y)}$$

(%i30) is(ratsimp(%));

(%o30) true

(%i31) W_:(m*q*(x-y))/(q*(x-y)*(1-m)+y)*c;

(%o31) $$\frac{c\,m\,q(x-y)}{y+(1-m)q(x-y)}$$

(%i32) w-W= -((1-m)*(x-y)*q+y)/((x-y)*q+y)*(W-W_);

(%o32) $$-\frac{m\,q\,y(W+c)}{y+q(x-y)}+\frac{m\,q\,x(W+c)}{y+q(x-y)}-W=\frac{(-y-(1-m)q(x-y))\left(W-\frac{c\,m\,q(x-y)}{y+(1-m)q(x-y)}\right)}{y+q(x-y)}$$

(%i33) is(ratsimp(%));

(%o33) true

/*

Inequality (13)

(%i34) kill(W,p);

(%o34) done

/*

Condition is given wrongly in the manuscript.
It should relate to mediocre productivity y
rather than prolific productivity x and read

py > W

and inequalities (13) should read

y > qxm/(1+qm) and mq < y/(x-y)

Replacing the inequality with equality,
I check whether these inequalities are equivalent to py > W.

(%i35)
```
p_:c/(y+q*(x-y)*(1-m));
```

(%o35)
$$\frac{c}{y+(1-m)q(x-y)}$$

(%i36)
```
solve([p_*y=W_],y);
```

(%o36)
$$[\, y=\frac{m\,q\,x}{m\,q+1}\,]$$

(%i37)
```
solve(p_*y=W_,q);
```

(%o37)
$$[\, q=-\frac{y}{m\,y-m\,x}\,]$$

```
/*
```

Quality mix (14) and (15)

(%i38)
```
diff(W_, q);
```

(%o38)
$$\frac{c\,m(x-y)}{y+(1-m)q(x-y)}-\frac{c(1-m)m\,q(x-y)^2}{(y+(1-m)q(x-y))^2}$$

(%i39)
```
is(ratsimp(%=(c*m*(x-y)*y)/(y+(1-m)*q*(x-y))^2));
```

(%o39) *true*

(%i40)
```
diff(p_,q);
```

(%o40)
$$-\frac{c(1-m)(x-y)}{(y+(1-m)q(x-y))^2}$$

(%i41)
```
is(ratsimp(%=-(1/(q*(x-y)*(1-m)+y))^(2)*c*(x-y)*(1-m)));
```

(%o41) *true*

```
(%i42) (c*q*(x-y)*(q*(x-y)+y)*m)/((c+W)*(y-q*(x-y)*(1-m)));
```

$$(\%o42)\quad \frac{c\,m\,q\,(x-y)\,(y+q\,(x-y))}{(y-(1-m)\,q\,(x-y))\,(W+c)}$$

```
/*

/*
*****APPENDIX: Performance Pay*****;

(%i43) kill(p,W,w);
(%o43) done
/*
The profit function (17) is

(%i44) prof(w,p):=p*(r*x+(1-r)*y)-r*w-(1-r)*(y/x)*w-c;
```

$$(\%o44)\quad prof(w,p):=p\,(r\,x+(1-r)\,y)-r\,w+(-(1-r))\,\frac{y}{x}\,w-c$$

```
(%i45) r:q*(1+m*log(w/W));
```

$$(\%o45)\quad q\left(m\,\log\left(\frac{w}{W}\right)+1\right)$$

```
(%i46) prof(w,p);
```

$$(\%o46)\quad p\left(y\left(1-q\left(m\,\log\left(\frac{w}{W}\right)+1\right)\right)+q\,x\left(m\,\log\left(\frac{w}{W}\right)+1\right)\right)+\frac{w\,y\left(q\left(m\,\log\left(\frac{w}{W}\right)+1\right)-1\right)}{x}-q\,w\left(m\,\log\left(\frac{w}{W}\right)+1\right)-c$$

```
/*
The derivative of prof(w,p) with respect to w is

(%i47) diff(prof(w,p),w);
```

$$(\%o47)\quad \frac{y\left(q\left(m\,\log\left(\frac{w}{W}\right)+1\right)-1\right)}{x}-q\left(m\,\log\left(\frac{w}{W}\right)+1\right)+p\left(\frac{m\,q\,x}{w}-\frac{m\,q\,y}{w}\right)+\frac{m\,q\,y}{x}-m\,q$$

```
(%i48) Dprof(w,p):=%o47;
```

$$(\%o48)\quad Dprof(w,p):=\%o47$$

```
(%i49) Dprof(w,p);
```

(%o49) $$\frac{y\left(q\left(m\,log\left(\frac{w}{W}\right)+1\right)-1\right)}{x}-q\left(m\,log\left(\frac{w}{W}\right)+1\right)+p\left(\frac{m\,q\,x}{w}-\frac{m\,q\,y}{w}\right)+\frac{m\,q\,y}{x}-m\,q$$

```
(%i50) W:w$
```

```
(%i51) solve([prof(w,p)=0, Dprof(w,p)=0],[w,p]);
```

(%o51) $$\left[\left[w=-\frac{c\,m\,q\,x\,y-c\,m\,q\,x^2}{(q^2-2\,q+1)\,y^2+(2\,q-2\,q^2)\,x\,y+q^2\,x^2},\ p=-\frac{(c\,(m+1)\,q-c)\,y+c\,(-m-1)\,q\,x}{(q^2-2\,q+1)\,y^2+(2\,q-2\,q^2)\,x\,y+q^2\,x^2}\right]\right]$$

```
/*
Check that expression (18) for w is correct
```

```
(%i52)
-(c*m*q*x*y-c*m*q*x^2)/((q^2-2*q+1)*y^2+(2*q-2*q^2)*x*y+q^2*x^2)
=(c*m*q*x*(x-y))/((1-q)*y+q*x)^2;
```

(%o52) $$\frac{c\,m\,q\,x^2-c\,m\,q\,x\,y}{(q^2-2\,q+1)\,y^2+(2\,q-2\,q^2)\,x\,y+q^2\,x^2}=\frac{c\,m\,q\,x\,(x-y)}{((1-q)\,y+q\,x)^2}$$

```
(%i53) is(ratsimp(%));
```

(%o53) *true*

```
/*
Compare flat pay with performance pay
```

```
(%i54) kill(W);
```

(%o54) *done*

```
(%i55) W_t:c*q*x*m*((x-y)/(q*x+(1-q)*y)^(2));
```

(%o55) $$\frac{c\,m\,q\,x\,(x-y)}{((1-q)\,y+q\,x)^2}$$

```
(%i56) V_t:(y/x)*W_t;
```

(%o56) $$\frac{c\,m\,q\,(x-y)\,y}{((1-q)\,y+q\,x)^2}$$

```
(%i57) W_:((m*q*(x-y))/(q*(x-y)*(1-m)+y))*c;
```

(%o57) $$\frac{c\,m\,q\,(x-y)}{y+(1-m)\,q\,(x-y)}$$

```
(%i58) (W_-(q*W_t+(1-q)*V_t))=
       (c*q^(2)*(x-y)^(2)*m^(2))/((q*x+(1-q)*y)*(q*(x-y)*(1-m)+y));
```

$$(\%o58)\quad -\frac{c\,m(1-q)q(x-y)y}{((1-q)y+q\,x)^2}-\frac{c\,m\,q^2x(x-y)}{((1-q)y+q\,x)^2}+\frac{c\,m\,q(x-y)}{y+(1-m)q(x-y)}=\frac{c\,m^2\,q^2(x-y)^2}{(y+(1-m)q(x-y))((1-q)y+q\,x)}$$

```
(%i59) is(ratsimp(%));
```

$$(\%o59)\quad true$$

/*

Wage Compression

```
(%i60) kill(all);
```

$$(\%o0)\quad done$$

/*

The Lagrangean is

```
(%i1) L(w,v,l):=q*(1+m*log(w/W))*(p*x-w)+(1-q)*(1+m*log(v/V))*
      (p*y-v)-c-l*(q*log(w/W)+(1-q)*log(v/V));
```

$$(\%o1)\quad L(w,v,l):=q\left(1+m\log\left(\frac{w}{W}\right)\right)(p\,x-w)+(1-q)\left(1+m\log\left(\frac{v}{V}\right)\right)(p\,y-v)-c+(-l)\left(q\log\left(\frac{w}{W}\right)+(1-q)\log\left(\frac{v}{V}\right)\right)$$

/*

The derivatives are

```
(%i2) L(w,v,l);
```

$$(\%o2)\quad -l\left(q\log\left(\frac{w}{W}\right)+(1-q)\log\left(\frac{v}{V}\right)\right)+q\,(p\,x-w)\left(m\log\left(\frac{w}{W}\right)+1\right)+(1-q)(p\,y-v)\left(m\log\left(\frac{v}{V}\right)+1\right)-c$$

```
(%i3) diff(L(w,v,l),w);
```

$$(\%o3)\quad -q\left(m\log\left(\frac{w}{W}\right)+1\right)+\frac{m\,q(p\,x-w)}{w}-\frac{l\,q}{w}$$

```
(%i4) DLw(w,v,l):=%o3;
```

$$(\%o4)\quad DLw(w,v,l):=\%o3$$

(%i5) `diff(L(w,v,l),v);`

(%o5) $$-(1-q)\left(m\,log\left(\frac{v}{V}\right)+1\right)+\frac{m(1-q)(p\,y-v)}{v}-\frac{l(1-q)}{v}$$

(%i6) `DLv(w,v,l):=%o5;`

(%o6) $DLv(w,v,l):=\%o5$

```
/*
We evaluate these expressions at W = w and V = v
```

(%i7) `W:w;`

(%o7) w

(%i8) `V:v;`

(%o8) v

(%i9) `solve([DLw(w,v,l)=0], w);`

(%o9) $$[\,w=\frac{m\,p\,x-l}{m+1}\,]$$

(%i10) `w_:(m*p*x-l)/(m+1);`

(%o10) $$\frac{m\,p\,x-l}{m+1}$$

(%i11) `solve([DLv(w,v,l)=0], v);`

(%o11) $$[\,v=\frac{m\,p\,y-l}{m+1}\,]$$

(%i12) `v_:(m*p*y-l)/(m+1);`

(%o12) $$\frac{m\,p\,y-l}{m+1}$$

(%i13) `ratsimp(w_-v_)=(p*(x-y)*m)/(1+m);`

(%o13) $$-\frac{m\,p\,y-m\,p\,x}{m+1}=\frac{m\,p(x-y)}{m+1}$$

(%i14) `is(ratsimp(%));`

(%o14) $true$

(%i15)