

Estimating Time-Varying Coefficients With the VC Program

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The object of a model is to segregate the semipermanent or relatively constant factors from those which are transitory or fluctuating so as to develop a logical way of thinking about the latter, and of understanding the time sequences to which they give rise in particular cases.

JOHN MAYNARD KEYNES (1978, 296)

1 Introduction

The estimation of models with time-varying coefficients is usually considered an arcane affair, and practical implementation requires much expertise and manual fine-tuning. The VC program described in this note is intended to improve the situation. It offers statistically superior alternative to Kalman-Bucy-Filtering, is easy to use, and freely available.

The following sections restate the underlying methodological considerations briefly, (Sect. 2), explain the statistical model and the estimation procedure (Sect. 3), and give a brief description of the program and its features (Sect. 4). A concluding note offers comments on some more advanced technical details (Section 5).

2 The Methodological Foundation: The Logic of *Ceteris Paribus* Analysis

Economic analysis proceeds *ceteris paribus*: The data of any problem - those things we presuppose in our argument in order to explain the behavior of some economic variables - are assumed constant, yet constant only for the time being. The assumption makes it possible to analyze selected issues in isolation and derive results that remain valid even if the factors frozen under a *ceteris paribus* clause are changing over time.¹

The main point in economic analysis seems to be that the data of any model must be *relatively* stable with regard to the processes under study, but it is neither necessary nor realistic to assume *absolute* stability here. Yet standard econometric models start from the premise of time-invariant coefficients. The more appropriate way is to estimate models with time-varying coefficients. The VC program is designed for the purpose of estimating time-varying parameter models. The classical regression model, which presupposes constant coefficients, is just a special case and may come about because the data say so. We need not assume stability from the outset, though.

¹See Schlicht(1978; 1997) for a mathematical statement and Schlicht (1977; 1985) for a comprehensive discussion of *ceteris paribus* analysis and a justification of using models with time-varying coefficients.

The assumption made here is that the behavioral data of the model move randomly over time, but move *relatively* slowly, as compared to speed of the endogenous variables. This means that the coefficients must be auto-correlated. As we do not want to explain the wanderings of these data of our analysis, because they are data, it seems sensible to assume a random walk here. This assumption assures that all changes in the coefficients are data-driven, rather than induced by the structure of the model. The assumption which really enters and links the past with the future - and we need an assumption of this kind - is the that the stability, or variance, of any coefficient is time-invariant, and we need to estimate this stability in order to draw conclusions about the parameters of interest.²

3 The Model

3.1 A Model With Coefficients Generated by a Random Walk

The Regression model considered in the following is

$$(1) \quad y_t = x'_t \cdot a_t + u_t, \quad t = 1, 2, \dots, T, \quad u_t \sim N(0, \sigma^2), \quad y_t \in \mathbb{R}, \quad x_t \in \mathbb{R}^n$$

with y_t and x_t denoting the observations at time t and T denoting the number of time periods covered by the observations. All vectors are taken as column vectors unless a transposition is indicated by an apostrophe "'". The vector $y' = (y_1, y_2, \dots, y_T)$ denotes the observations of the dependent variable. The vector $x'_t = (x_{1,t}, x_{2,t}, \dots, x_{n,t})$ denotes the vector of the n independent variables observed at time t . The disturbance u_t is assumed to be i.i.d. normally distributed with mean zero and variance σ^2 .

The coefficients a_t in equation (1) are assumed to be generated by a random walk:

$$(2) \quad a_t = a_{t-1} + v_t, \quad t = 2, 3, \dots, T, \quad v_t \sim N(0, \Sigma), \quad a_t \in \mathbb{R}^n$$

with

$$\Sigma = \begin{pmatrix} \sigma_1^2 & & & 0 \\ & \sigma_2^2 & & \\ & & \ddots & \\ 0 & & & \sigma_n^2 \end{pmatrix}$$

The classical regression model is covered as the limiting case $\Sigma \rightarrow 0$ where the coefficients remain invariant over time. The case of interest is that the coefficients move slowly over time (that Σ is small), but in the end the volatility of the coefficients is to be determined empirically rather than assumed in advance.

²A detailed elaboration of the underlying "Isolation Principle" and its entailments for econometrics is provided in Schlicht(1985).

The model (1, 2) can be reformulated in matrix notation as

$$(3) \quad y = X \cdot a + u, \quad u \sim N(0, \sigma^2 I)$$

$$(4) \quad P \cdot a = v, \quad v \sim N(0, S), \quad S = I \otimes \Sigma$$

with

$$X = \begin{pmatrix} x'_1 & & & 0 \\ & x'_2 & & \\ & & \ddots & \\ 0 & & & x'_T \end{pmatrix}, \quad a = \begin{pmatrix} a_1 \\ a_2 \\ \vdots \\ a_T \end{pmatrix}$$

and

$$P = \begin{pmatrix} -I & I & & & 0 \\ & -I & I & & \\ & & -I & I & \\ 0 & & & -I & I \end{pmatrix}$$

3.2 Estimation

For given observations y and X and given σ^2 and Σ , the conditional distribution of the coefficients a is normal with expectation

$$(5) \quad \hat{a} = (X'X + \sigma^2 P' S^{-1} P)^{-1} X'y$$

(see Schlicht 1985; 1988). The variances can be estimated by a generalized method of moments (GMM) procedure and plugged into equation 5 (see Schlicht 1988). The program VC performs these necessary calculations.

The covariance matrix of the deviations of the estimates from the true coefficients $(\hat{a} - a)$ can be obtained from Schlicht(1988, eq., 40) as

$$(6) \quad E((\hat{a} - a)(\hat{a} - a)') = \sigma^2 (X'X + \sigma^2 P' S^{-1} P)^{-1}$$

The square roots of the elements of the main diagonal are given in the program as well. As an option, the entire matrix (6) is calculated.

4 The Program

If the program is called, a window opens that displays a number of options and controls. On these later. The main buttons are "Get input!," "Info," and "Cancel." The "Cancel" button stops the program, the "Info" button opens an information file in PDF format, and the "Get input!" button starts the program. A window opens that allows for selecting an input file.

4.1 Input

The program requires input data in the .CSV format (comma separated values).³ An example input file (input.csv) is provided. The first row is reserved for data description - one for each cell. No cell should be left empty. The number of cells in the first row tells the program how many cells to expect in the subsequent rows. The second row gives the starting values for variances. Iteration starts from these variances. The subsequent rows give the data::

Each row contains

-cell 1: time index, such a year or month, ignored by the program, but must be non-empty and can't contain a comma.

-cell 2: dependent variable (typically $y(t)$)

-cell 3: first independent variable ($x_0(t)$, typically constant term, =1)

-cell 4: second independent variable ($x_1(t)$)

-cell 5: third independent variable ($x_2(t)$).

-etc.

4.2 Output

If the input file is OK, the iteration starts. The number of iterations is displayed in a window, as well as the achieved precision (measured by the maximum relative deviation of expected from theoretical variances). If a very high precision is reached or if non-computable values occur, the program stops. If the program is interrupted (because precision is good enough, for instance), or stops on its own, it asks for the name of the output file. This file will be written in .csv format. as well

It will contain the original data and the calculated coefficients together with their standard errors in additional columns.

At the end a list of the initial variances and the computed variances are given.

-cell 1: Variance of disturbance in the equation

-cell 2: variance of the first coefficient

-cell 3: variance of the second coefficient

³Note that the English number format is presupposed, with a decimal dot denoting decimals, and a comma separating values. Other formats (the German one, for instance) use other conventions and need to be transformed accordingly.

etc...

4.3 Options

The program allows for four options:

Estimate all variances. This is the standard case. If this is selected, all variances are estimated.

Estimate selected variances only. In this case a new window opens after the input data is read. Here you can force selected variances to the initial value given in the input file and adjust only the other variances. The output file will contain information on the enforced variances and on the corresponding empirical variances in this case.

Don't estimate variances. In this case, the coefficients are computed for the variances given in the input file according to eq. 5. The corresponding empirical variances are given in the output file.

Calculate complete covariance matrix. If this is selected, the program will ask for a further output file where it will save the covariance matrix (6).

4.4 Controls

Two checkboxes are provided:

Slow down adjustment. This slows down the adjustment during iterations and may help to cope with some stability issues. This option has been introduced just as a precaution, as the experience with the numerical performance of the program is still slim.

Iterate until aborted. This option will disable automatic termination of the iteration process. You need to close the info window in order to terminate the iteration process in this case.

5 A Note on Kalman-Bucy Filtering

It is usual to employ Kalman-Bucy filtering in order to estimate models with time-varying coefficients like (1), (2). I have proposed this technique myself some time ago (Schlicht 1977), but have convinced myself that the estimation technique described in Schlicht(1988) and implemented in the VC program here is preferable because it is statistically more efficient and numerically more transparent and straightforward. The main issue in both Kalman-Bucy filtering and in my approach relates to the estimation of variances. But given the variances, the VC program is clearly statistically superior because it is, by construction, a one-sided filter. The estimate of the state of coefficients a_t at time t will depend only on past and current, but not on future observations - that is, on $(y_\tau, X_\tau)_{\tau=1,2,\dots,t}$. But the current state a_t is correlated with the future states, and future observations will provide information about these states. Hence it is inefficient to use a one-sided filter. The method

implemented in VC uses all observations for estimating each coefficient that is, on $(y_\tau, X_\tau)_{\tau=1,2,\dots,T}$

The second issue relates to the estimation of variances. This is usually done by maximum-likelihood methods in the Kalman-Bucy framework. It can be done in my model as well; it is, in fact, much more easy to implement. My experience with maximum-likelihood estimation for smaller samples was, however, entirely disappointing, and so I developed the GMM estimator which seems to work much better in smaller samples, has a nice intuitive interpretation and is asymptotically equivalent to the maximum-likelihood estimator. A comparably transparent and computationally manageable method seems not available for Kalman-Bucy filters.

6 Terms and Conditions

All rights reserved. This program is free for non-commercial use. It may be freely used and distributed provided that all the original files are included - in particular this info file. The program is provided as-is, without any warranty or obligation on my part. The user is invited to improve the program. If he distributes his own version, he is required to include a copyright notice which states how the derived program is related to the original one. The distribution of the derived program must include this original distribution.

I would appreciate comments, suggestions, bug reports, and observation on overall performance at schlicht@lmu.de

The most recent version is available on my website www.lrz.de/~ekkehart -> downloads

7 History

March 6, 2003 version 3 with covariance option.

Feb. 26, 2003, version 3.0 beta with new GUI and extended information.

Feb. 12, 2003 version 2.0, improved by Robert Schlicht, outputs variances, additions to Readme.txt.

Feb. 7, 2003 addition to Readme.txt

Feb. 1, 2003 first public release, ver. 1.0

Dec. 2002 translation of the original code into C by Robert Schlicht

Sept. 25, 1987 original program written in an obsolete dialect by Ekkehart Schlicht

8 Acknowledgements

Robert Schlicht has ported the original program into C and wrote the input-output modules.

The data for the test file (input.csv) are taken from Manfred Gaertner's website <http://www.fgn.unisg.ch/eumacro/macrodata/dmtrxneu.htm>.

Johannes Ludsteck suggested the handling of $E((a - \hat{a})(a - \hat{a})')$.

9 References

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