On Keynesian Supply

by

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0. Introduction

As Klein has noted recently, an explicit consideration of the supply side within the framework of macroeconomic theory seems to be desirable in order to enhance our understanding of many of today's economic problems[1]. Keynes himself, however, offered a treatment of aggregate supply in the General Theory, which has been conceiled by the reformulation of the Keynesian model due to Hicks and Hansen and by some subsequent semantic confusion[2].

The purpose of the following remarks is to help clear this confusion and to show that Keynes offered a treatment of aggregate supply which might provide an interesting alternative to the way those problems have been tackled by his successors. Furthermore, the discussion might shed some light on the relationship of Keynes' thinking to recent "fixprice" or " quantities-faster-than-prices" conceptions.

1. Semantic Confusion

When Keynes introduced the concept of Effective Demand, he did so by defining it as the point of intersection between the aggregate supply function and the aggregate demand function[3]. The new staging of this theory by Hicks avoided all of these three concepts, but later writers, starting from the Hicksian scheme, have identified the notions of effective demand and aggregate demand and dropped the concept of aggregate supply altogether[4].

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1) Klein (1978)
2) Keynes (1936), Hicks (1937), Hansen (1953)
3) Keynes (1936, pp. 25,55)
4) Hicks (1937) The new choice of units introduced here (taking income rather than employment as the determinant of aggregate demand and aggregate supply) led Hansen, in his most influential interpretation of Keynes, to identify the 45°-degree line with the aggregate supply function, see Hansen (1953, p.31). By this, "all the stuffing, so to speak, has been taken out of it before it is allowed to appear in the diagram"; as Robertson (1955, p.475) put it. As Schneider (1965, p.120n) has remarked, the 45°-line technique has been introduced by the Danish author F. Jantzen in 1935, i.e. *before* the General Theory has been published. It has been

Even specialised works dealing exclusively with the exegesis of Keynes' writings in a very scholary manner, like Leijonhufvuds, provide no exception here, and Klein's otherwise rather sanguine statement: "It is well known that Keynes included the aggregate supply function in the General Theory, but it was introduced in his chapter on 'The Principle of Effective Demand'"[1] reveals the same semantic flaw. Semantic problems became aggravated, furthermore, when Patinkin introduced another kind of function which he nevertheless called a "Keynesian Supply Function"[2]. Patinkin developed this concept to what might be called the "Patinkin Supply Function"[3]

As far as the supply side in short run macroeconomic analysis is considered at all, this Patinkin aggregate supply function has been usually employed[4]. It is defined simply as that level of output which maximises profits at the ruling real wage rate[5].

introduced by Klein (1947, p.115) as the "entire cornerstone of the Keynesian System". Whereas Dillard (1948, p.35)- and Vandenborre (1958, p.203,216) later on - has stressed that the 45°-line is different from the Keynesian supply function, this distinction has been abandoned by Hansen (1953, p.31). Edwards (1959, p.416) has tried to give a reinterpretation of the 45°-line as a supply function, however. See also De Jong (1954(1)).

1) Leijonhufvud (1968), Klein (1978), quotation on p.2. It is remarkable that an author of this distinction, who has published an important book on the very topic of Keynesianism, could have fallen into this semantic trap. From today's point of view it might have been better if Keynes had replaced his notion of effective demand by another word, e.g. effective equilibrium or whatever you like, but avoiding the word "demand". This caused trouble from the beginning. Hawtrey (1954) e.g. maintains: "But for Keynes effective demand is the single determinant" (p.834) or "Keynes dispensed with the familiar supply function"(p.837). There is sufficient evidence as to the contrary, however; see Robertson (1955, p.474-5) or De Jong (1954(1)), for instance.

2) Patinkin (1949) The difference to the Keynesian concept has been noted by Patinkin himself, see Patinkin (1965, pp.666-7): He seems to blame De Jong for having pointed it out unneccessarily.

3) Patinkin (1965, p.210)

4) by Malinvaud (1977) for instance

5) Denote by v the real wage rate and by $X = F(N)$ the aggregate production function (capital stock fixed). The Patinkin supply function $S(v)$ is defined implicitely by the marginal condition $F' = v$, i.e.

$$S(v) := F\{F'^{-1}(v)\}$$

see Patinkin (1965, p.210)

There is one stream of theorizing, however, which is much closer to Keynes than mainstream Keynesianism, and which is associated with the name of Sidney Weintraub[1]. Yet his exposition, like that of Keynes, does not explicitly start from a production function to explain supply, but casts the theory from the beginning into aggregates comprising quantities and prices (i.e. the Keynesian 'proceeds') which change simultaneously as employment changes. This line of procedure has proven to be somewhat difficult to understand from the point of view of the modern economist, and it leaves the real underlying processes somewhat unexplained.

The following remarks are intended to circumvent these difficulties An attempt is made to explain the Keynesian Supply Function, and it is endeavoured to explain it not in Keynesian terms but in terms of notions usually employed in standard macro models, namely

1.) by confining the analysis to a one-sector-model where production can be explained by an aggregate production function (rather than considering a two sector model[2])
2.) by measuring aggregates in physical terms rather than in wage units and introducing wages and the price level explicitely
3.) by taking real income, rather than employment, as determining aggregate supply and aggregate demand.

This kind of analysis might be understood, therefore, as being an introduction to Keynes's somewhat more complex and much more sophisticated analysis which makes it possible to contrast standard models - like Patinkin's - with some features of the Keynesian vision.

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1) Weintraub (1956,1957) and, for a more recent exposition of his theories, Weintraub (1966). A very clear statement can also be found in ch. III of Davidson and Smolensky (1965) Weintraub's contribution seems to have been somewhat neglected in the literature. The most influential works by Leijonhufvud or Coddington, for example, do not mention it.
2) or following Keynes' multi-sectorial remarks in chapter 20 of the General Theory. (Keynes 1936, pp. 280-286) See Vandenborre (1958,pp.205-210) for an attempt in that direction.

2. The Aggregate Supply Function: Basic Analysis

Assume the short-run production function

(1) $X = F(N), \quad F' > 0, \quad F'' < 0$

relating output X to employment N. For a given price level of output p and a given wage level w, profit maximising employment is given by the marginal condition

(2) $p\,F'(N) = w$

For a given level of employment N and a given level of wages w, competition forces the price level to the unique value satisfying (2)[1] This does not require, of course, that the wage level be fixed - it is assumed that this holds true for any given wage level[2]. Thus equation (2) can be viewed as determining the real wage level w/p.

Aggregate supply Y^S is defined by Keynes as the value of output belonging to a given level of employment, i.e. he writes[3]

(3) $Y^S = p \cdot X = w \cdot \frac{F(N)}{F'(N)} =: w \cdot \phi(N)$

Since Keynes' analysis proceeds in wage units, he takes $w = 1$ and denotes $\phi(N)$ as the aggregate supply function.

Equivalently we can assume the wage level to be arbitrarily fixed without affecting the results of the analysis - which remain valid in the case of flexible wages, since the wage level serves merely as a numéraire.

Using the production function which relates production to employment, we derive the aggregate supply function relating production to the value of aggregate demand.

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1) cf. Keynes (1936, pp. 12,17)
2. cf. Keynes (1936, pp. 269-271). Thus it is assumed that *prices are more flexible than wages* - an assumption which might be disputed on empirical grounds.
3) Keynes (1936, p.25). Keynes denotes aggregate supply in wage units by Z, i.e. $Z = Y^S/w = \phi(N)$

(4) $Y^S = w\varphi(X)$

where $\varphi(X) = \frac{F(F^{-1}(X))}{F'(F^{-1}(X))} = \frac{X}{F'(F^{-1}(X))}$, $\varphi' = ((F')^2 - X \cdot F'')/(F')^3 > 0$ /

Aggregate supply is an increasing function of production which approaches infinity as the marginal productivity F' approaches zero.

For purpose of illustration- and because it seems to be economically sensible - it will be assumed in the following that production cannot be increased beyond a certain limit $\bar{X}$, as determined by the existing capital stock, even if employment could be increased indefinitely[1].

(5) $F(N) \leq \bar{X}$ for all $N \geq 0$

This implies that the aggregate supply function $\varphi(X)$ approaches infinity if X approaches $\bar{X}$ from below. (Fig. 1.)

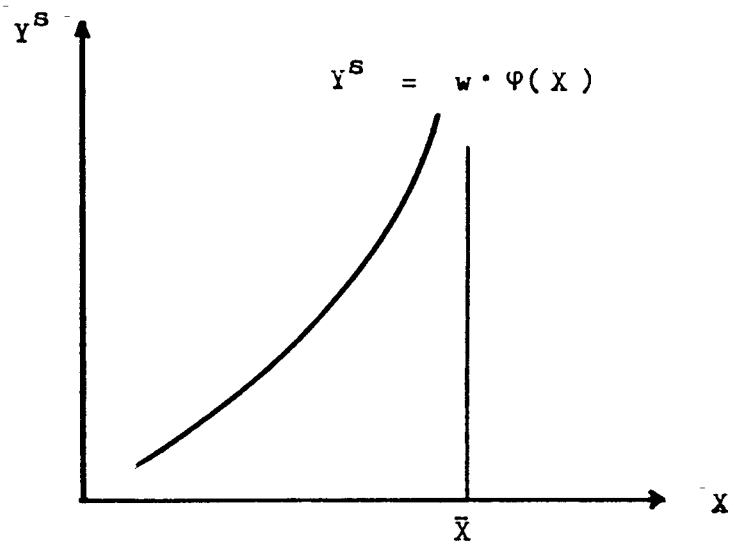

Figure 1

The Aggregate Supply Function φ relating real production X to monetary supply Y^S for a given wage level w

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1) The CES-production function

$$X = \beta \cdot \{a \cdot N^{\frac{\sigma-1}{\sigma}} + (1-a)K^{\frac{\sigma-1}{\sigma}}\}^{\frac{\sigma}{\sigma-1}}$$

implies

$$\bar{X} = \beta \cdot (1-a)^{\frac{\sigma}{\sigma-1}} \cdot K$$

as long as the elasticity of substitution σ is less than unity, e.g.

To conclude this basic analysis of aggregate supply, the Keynesian definition shall be given, albeit translated into more familiar terms: The aggregate monetary supply Y^s belonging to a given amount of production X is that amount of monetary demand which will just make it worth while of the entrepreneurs neither to expand nor to reduce production[1]. The analysis given up to now is intended to explain this definition in a rather special model. Its final relevance will be elucidated to a somewhat larger extend in section 4. But before this is done, some remarks on the interaction of aggregate demand and aggregate supply seem to be in place.

3. Aggregate Supply and Aggregate Demand as Determining the Level of Effective Demand

The purpose of these remarks being not to explore the Keynesian concept of aggregate demand in the same detail as that of aggregate supply, it seems nevertheless appropiate to indicate roughly the use of the aggregate supply function. This can only be done with reference to aggregate demand, however.

Since a detailed discussion of Keynesian aggregate demand shall be avoided, the rough treatment will be somewhat less faithful to Keynes than the discussion of aggregate supply has been[2].

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1) The original reads: "On the other hand, the aggregate supply price of the output of a given amount of employment is the expectation of proceeds which will just make it worth which of the entrepreneurs to give that employment" (Keynes (1936)p. 24). Note that the definition given in the text is derived from that definition by substituting the level of employment by the corresponding level of production and by replacing expected demand by actual demand - the Keynesian definition is much more general, therefore.

2) Vandenborre (1958) has developed the Keynesian theory of aggregate demand by distinguishing between monetary demand proper and demand as seen by the sellers. This distinction has not been made sufficiently clear by Keynes himself and lead to criticisms (Hawtrey (1956), Robertson (1956)). The above presentation follows Keynes in that respect although Vandenborre's presentation might be more appropriate.

Any hypothetical level of employment N gives rise to a certain wage bill $w \cdot N$ and corresponding real production $X = F(N)$. If the entrepreneurs decided to realize this level of employment, they would only do so, if monetary demand where equal to the corresponding level of aggregate supply $Y^s = w \cdot \varphi(X)$. Else they would chose a larger or a smaller level of production, respectively.

The aggregate demand belonging to a given level of production X is derived from the hypothetical assumption that the entrepreneurs can sell this production at the price Y^s/X such that their proceeds are equal to the corresponding aggregate supply $Y^s = w\varphi(X)$. Since the wage bill is $w \cdot N = w \cdot F^{-1}(X)$, gross profits are $w\varphi(X) - w \cdot F^{-1}(X)$. Furthermore, the price level is fixed by equation (2), and we can write monetary consumption demand as

$$(5) \quad C = C(\underbrace{wF^{-1}(X)}_{\text{wage bill}}, \underbrace{w \cdot \varphi(X) - wF^{-1}(X)}_{\text{gross profits}}, \underbrace{w/F'(F^{-1}(X))}_{\text{price level}})$$

If it is assumed that there is no money illusion, this function will be homogeneous of degree one and we can take

$$G = w \cdot c(X)$$

as our consumption function[1]. If the rate of interest is taken to be given, or to be uniquely correlated with the level of production X, we might derive in a similar fashion an investment demand function, e.g.

$$(7) \quad I = I(\underbrace{\bar{X} - X}_{\substack{\text{excess} \\ \text{capacity}}}, \underbrace{r(X)}_{\substack{\text{rate of} \\ \text{interest}}}, \underbrace{w\varphi(X) - F^{-1}(X)}_{\text{gross profits}}, \underbrace{w/F'(F^{-1}(X)}_{\text{price level}}, \text{..expectations})$$

boiling down to

$$I = w \cdot i(X)$$

---

1) From this it can be seen that distributional effects are included in the Keynesian consumption function. This has been stressed by Keynes quite forcefully, see Keynes (1973, pp.270-277; 1936, p.96). Under this point of view, Kaldor's theory of distribution is not a Keynesian one whereas Weintraub's is, see Kaldor (1956, pp. 94-100), Weintraub (1956, 1966 ch. vii) See also Marty (1961) for a beautyful geometrical exposition of the Keynesian theory of distribution.

The sum of consumption demand (6) investment demand (8) gives rise to the aggregate demand function

(9) $Y^D = w\ f(X)$

Let it be assumed without further analysis that this aggregate demand function has the shape as indicated in Fig. 2, where it is plotted together with the aggregate supply function:

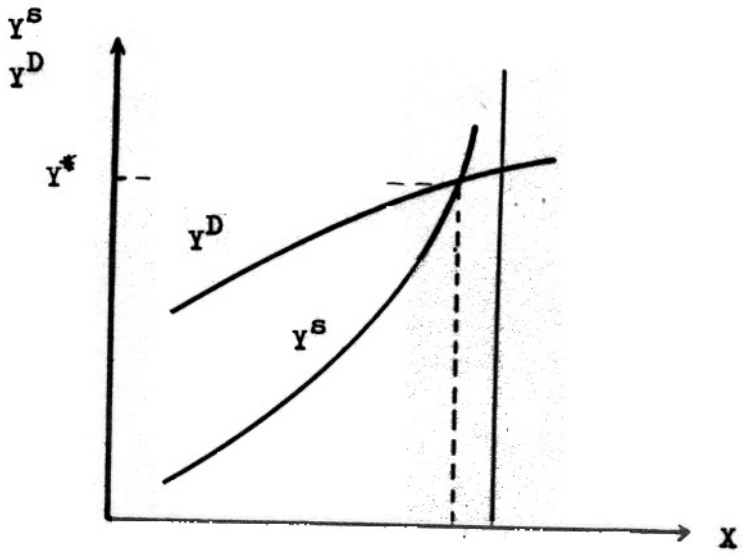

Figure 2

Aggregate Demand Y^D and Aggregate Supply Y^S as determining the point of Effective Demand Y^*

This figure looks very similar to the usual 45^{o}-degree-line diagram, where the 45^{o}-degree line is replaced by the Keynesian supply function and the analysis proceeds in a similar fashion[1].

If production is below its equilibrium value X^*, the entrepreneurs could realize income Y^S and even more since there is excess demand ($Y^D > Y^S$): they could charge higher prices and increase production up to the point X^*.

On the other hand, if production where above X^*, where aggregate demand Y^D is below Y^S: entrepreneurs could not realize Y^S, therefore they could not even realize Y^D since Y^D is construed under the assumption that Y^S be realized!-. This will induce a reduction of output up to the point where X^* is reached.

The level Y^* of demand corresponding to X^* is termed *Effective Demand*, and this is the point which is realised by the above-mentioned process.

Note that close to the full capacity constraint $\bar{X}$, there might be considerable unemployment in the usual sense of the term. For example, insufficient accumulation in the past or too capital-intensive investment might have led to a number of jobs available in the economy which falls short of the number of jobs required by those seeking employment. Nevertheless, this situation is characterised, according to Keynes' definition, by the absence of involuntary unemployment - he explicitely <u>defines</u> it as one of full employment ![2] So-called Keynesian policy measures would <u>not</u> be recommended by Keynes' General Theory under these circumstances.

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1) see p.1 note 4 above on the 45^{o} - line analysis

2) Keynes (1936), pp. 27,15

4. The Concept of Aggregate Supply: Refinements

In the basic analysis of aggregate supply of section 2, some features of the aggregate supply function have been neglected which are quite important in Keynes' theory as well as in reality. Reflections on two of them follow.

The first pertains to *labour shortage*. The maximum supply $\bar{X}$ has been derived under the assumption of a sufficient labour supply, but this barrier might be much lower if there is labour shortage with respect to the capital stock. Let $\bar{N}$ be the maximum labour supply. The maximum supply forthcoming for any level of aggregate demand will now be given by[1)]

$$(10)\quad \bar{X} = \max \{F(N),\ N \leq \bar{N}\}$$

The resulting supply function is depicted in fig. 3

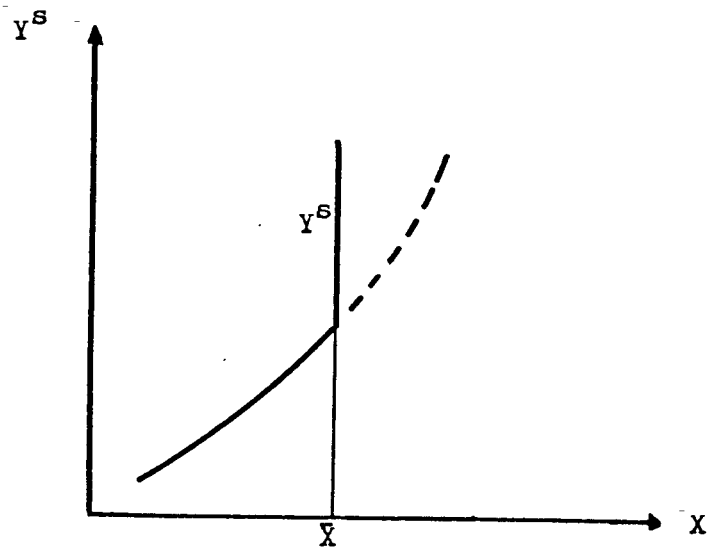

Figure 3

The Aggregate Supply Function in presence of a full employment constraint

1) N might in turn be dependent on the real or money wage level or the real wage level, cf. Keynes (1936, p.30). It is not too difficult to incorporate this assumption into the formal analysis by introducing the money wage level or the real wage level as a seperate argument into (10)

Furthermore, *bottlenecks* near full capacity can be taken into consideration, since all industries will not hit the capacity limit simultaneously, but successively[1]. Instead of fig. 3 the aggregate supply function a will look like the heavily drawn line in fig. 4

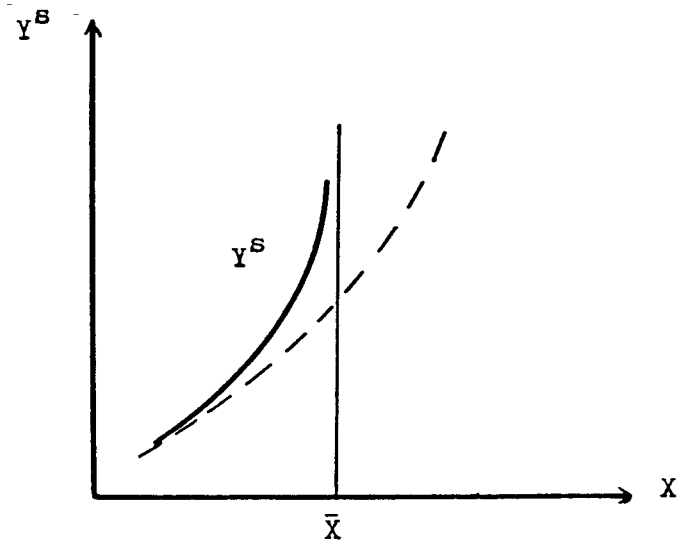

Figure 4

The Aggregate Supply Function taking account of bottlenecks near the full employment constraint

The shape of this curve is similar to that of the aggregate supply function which has been derived in section 3, but the interpretation of the function has been changed towards an incorporation of more realistic and practically important features.

1) see Keynes .(1936, p.300)

5. Concluding Comments

Although this kind of re-interpretation of the theoretical analysis, and of re-focussing it towards the incorporation and treatment of relevant economic phenomena, can be continued (and has been elaborated by Keynes to a considerable extend), a further theoretical development in this direction will not be attempted here. Nevertheless, the fruitfulness of any theoretical concept - like that of the aggregate supply function - hinges critically upon the scope it leaves for more detailed reinterpretation, i.e. the possible domain of relevant problems which can be handled by that particular analytical tool. Under this perspective, the Keynesian aggregate supply function seems to me to be of considerable analytical power and might deserve some more attention than it has received up to now. This does not mean, however, that the particular interpretation of this concept given in this paper - or even the derivation given by Keynes - has to be maintained[1)]. Rather it is the perspective, the general point of view, which characterises this concept. I feel the following tentative definition to come rather close to the gist of the matter:

> *For any particular level of employment, the Aggregate Supply Function gives that level of Aggregate Demand which will cause the entrepreneuers neither to expand nor to reduce production*[2)].

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1) E.e. the assumption incorporated in (2) might be abandoned in favour of pricing assumption resulting from imperfect competition- and similarily, any other building block might be replaced by something more appropriate for the particular problem at hand. As Keynes (1939) has conceded, observed movements of real wages are just opposite to the behaviour described by (2). He has pointed out that this seems to be even more favourable to his main conclusions,and he indicates as one solution to the problem that increasing returns are prevailing even in the short run, an assumption which has been reconfirmed by the observation that a one per cent change in employment is usually associated with a change in output of more than one per cent (Okun's law). If increasing returns are combined with a version of mark-up pricing (Harrod 1973, p.4-7) this can serve as a plausible alternative of (2), for instance.

As to the recent "disequilibrium" interpretation given to Keynes by Clower, Leijonhufvud, and Malinvaud, for example, it is to be noted, that the Keynesian view is quite different in that prices are assumed to be equilibrium prices according to (2), and no "false trading" is considered to be of relevance. Unemployment - in the view of these authors - is caused by information failures (to the absence of the auctioneer and to wrong prices: If prices were right, these problems would disappear. In contrast to that, Keynes has set out himself to explain unemployment in presence of *flexible* prices. It is to be doubted that he would have written a book on the thesis that wrong prices might produce uncleared markets[1)].

2) Symmetrically, the aggregate demand function has to be defined as follows: For any particular level of production, the aggregate demand function gives that amount of aggregate supply which will cause aggregate demand neither to expand nor to contract. This concept - being more complicated as that of the aggregate supply function, as noted by Weintraub (1957, p.455n) - would need some further elaboration, however, which is beyond the scope of this paper.
It is to be noted that the above definition of aggregate supply is very close to the Marshallian notion of supply: "So the price required to call forth the exertion necessary for producing any given amount of a commodity, may be called the *supply price* for that amount during the same time" (Marshall, p.118)

1) as has been remarked by Bliss (1975), p. 210. See Shackle (1973), Coddington (1976) and Schlicht (1978) on some aspects of the interpretation of Keynes. By the way, Arrow (1959) put foreward the purely logical proposition that you cannot have both price taking behaviour and flexible prices without somebody changing those prices, that is, an auctioneer. It seems to make for the charm of Leijonhufvuds work to try to circumvent this logical necessity.

References:

Arrow, K.J.	Toward a Theory of Price Adjustment in: M. Abramovitz (ed.): The Allocation of Economic Resources Stanford 1959
Bliss, C.	The Reappraisal of Keynes' Economics: An Appraisal in: Current Economic Problems, Proceedings of the AUTE, edited by M. Parking and A.R. Nobay, Cambridge: Cambridge University Press 1975, pp. 203-216
Clower, R.W.	The Keynesian Counter-Revolution: A Theoretical Appraisal in: F.H. Hahn und F.P.R. Brechling (eds) The Theory of Interest Rates London 1965
Coddington, A.	Keynesian Economics: The Search for First Principles in: Journal of Economic Literature 1976, pp. 1258-1273
Davidson, P.	Note on the Aggregate Supply Function in: Economic Journal 1962, pp. 452-457
Davidson P. and Smolensky E.	Aggregate Supply and Demand Analysis New York 1964
de Jong, F.J.	Supply Functions in Keynesian Economics in: Economic Journal 1954, p. 3-24
de Jong, F.J.	Keynes and Supply Functions: A Rejoinder in: Economic Journal 1954, pp. 840-42
de Jong, F.J.	Keynes and Supply Functions: Second Rejoinder, with a note on the Concept of Monetary Equilibrium Economic Journal 1955, pp. 479-484
de Jong, F.J.	Keynes and Supply Functions: Third Rejoinder and Final Observations Economic Journal 1956, pp. 488-492
Dillard, D.	The Economics of John Maynard Keynes New York 1948
Edwards, E.O.	Classical and Keynesian Employment Theories: A Reconciliation Quarterly Journal of Economics 1959, pp. 407-428

Hansen, A. — A Guide to Keynes
New York 1953

Harrod, R. — Economic Dynamics, London 1973

Hawtrey, R.G. — Keynes and Supply Functions
in: Economic Journal 1954, p. 834-839

Hawtrey, R.G. — Keynes and Supply Functions:
A Further Note
in: Economic Journal 1956, pp. 482-84

Hicks, J.R. — Mr. Keynes and the "Classics"
in: Critical Essays in Monetary Theory
Oxford UP. 1967 pp. 126-142
Reprinted from Econometrica vol. V No. 2
April 1937

Hotson, J.H. — Walras' General Equilibrium and the
Economics of Keynes in:
Economie Appliquée 1977, pp. 139-160

Jackman, R. — Keynes on Leijonhufvud
in: Oxford Economic Papers 1974, pp. 259-271

Johnson, H.G. — Mathematical Appendix to "Keynes on Supply
Functions" by D.H. Robertson
in: Economic Journal 1955, pp. 477-478

Kaldor, N. — Alternative Theories of Distribution
in: Review of Economic Studies 1956, pp.83-100

Keynes, J.M. — The General Theory of Employment, Interest
and Money
London 1936

Keynes, J.M. — Relative Movements of Real Wages and Output
in: Economic Journal 1939, pp. 34-51

Keynes, J.M. — The General Theory and After
Part II: Defense and Development
The Collected Writings of John Maynard Keynes
edited by D. Moggridge, Vol. XIV
London 1973

Klein, L.R. — The Keynesian Revolution
New York 1947

Klein, L.R. — The Supply Side
in: American Economic Review March 1978, pp.1-7

Leijonhufvud, A. — On Keynesian Economics and the Economics
of Keynes
London 1968

Malinvaud, E. — Theory of Unemployment Reconsidered
Oxford: 1977

Marty, A.L.	A Geometrical Exposition ofthe Keynesian Supply Function in: Economic Journal 1961, pp. 560-565
Marshall, A.	Principles of Economics Eighth Edition London: Macmillan 1920
Patinkin, D.	Involuntary Unemployment and the Keynesian Supply Function in: Economic Journal 1949, S. 360-83
Patinkin, D.	Money, Interest and Prices, 2nd ed. New York 1965
Robertson, D.H.	Keynes and Supply Function in: Economic Journal 1955, pp. 474-477
Robertson, D.H.	Keynes and Supply Functions in: Economic Journal 1956, pp. 885-87
Schlicht, E.	Note on Keynesianism in: Zeitschrift für die Gesamte Staatswissenschaft, 1979
Schneider, E.	Einführung in die Wirtschaftstheorie Vol. III, 8th edition Tübingen 1964
Shackle, G.L.S.	Keynes and Today's Establishment in Economic Theory: A View in: Journal of Economic Literature 1973, pp. 516-519
Vandenborre, H.	An Intergration of Employment Economics Within the Keynesian Theory of Money Flows Oxford Economic PapersJune 1958, pp. 205-19
Veendorp, E.C.H. and Werkema, H.G.	Mr. Wells' Aggregate Supply Function A Comment in: Economic Journal 1961, pp. 634-636
Weintraub, S.	A Macroeconomic Approach to the Theory of Wages American Economic Review Dec. 1956, S. 835-36
Weintraub, S.	The Micro-Foundations of Aggregate Demand and Supply in: Economic Journal 1957, pp. 455-70
Weintraub, S.	Employment, Growth, and Income Distribution Philadelphia: Chilton 1966
Wells, P.	Keynes Aggregate Supply Function: A Suggested Interpretation in: Economic Journal 1960, pp. 536-542
Wells, P.	A Further Comment in: Economic Journal 1961, pp. 636-37